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Numerical Solution for mixed linear Volterra-Fredholm integral and Integro-differential equations using Monic Chebyshev Polynomial

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ABSTRACT

In this paper, a numerical approach is developed for solving mixed linear Voltera-Fredholm integral and Integro-differential equations of the second kind. The approximate solution is substituted into the model equation and then collocated using Monic Chebyshev polynomial and Standard collocation points to obtain a system of linear algebraic equations, which is then solved by conjugate gradient method. Several numerical examples were solved to demonstrate the accuracy, reliability and efficiency of the method. The results obtained give an exact solution and were found to be accurate.

Keywords: Integro-differential equations, standard collocation points, monic Chebyshev polynomial, conjugate gradient method.

1.0 INTRODUCTION

The theory and application of integral and Integro-differential equations are important subject within applied mathematics Dasjerdi and Ghaini (2012). Numerical solutions of many integral and Integro-differential equations arise in mathematics modeling of many phenomena Biazar and Ebrahimi (2011). Volterra and Fredholm integral and Integro-differential equations has application in models of physical phenomena such as renewal theory, population dynamics, the spread of disease, and many fields of science and engineering Dasjerdi and Ghaini (2012), Hale (2017), Kuang (1993), Medlock and Kot (2003).

Numerical solutions of integral equations are studied by Abubakar and Taiwo (2014), Babolian, E., Masouri and HatamzadehVarmazyar (2018), Cardone, Conte Ambrosio and Paternoster (2018), Doaa (2019), Eshkuvatov, Z.K., Zulkarnain, Nik-Long and Muminiv (2016), Gegele, Evans and Akoh, (2014), Hassan and Sulaiman, (2018), Ibrahim, Attah and Gyegwe (2016), Jafar, Parviz and Ali (2012), Medlock and Kot (2003), Mustafa and Muhammad (2014), Nargess, Mahmoudand and Habibollah (2020), Nemati, (2015), Rasty, and Hadizadeh, (2010), Shahooth, (2015), Sohrab (2016), Yuzbasi and Karacayir (2020).

In this work, we extend the work of Agbolade and Anake (2017) to mixed linear integral and Integro-differential equations of m-*th* order of the form;

$$u^{(m)}(x) = g(x) + \int_0^x \int_a^b K(r,t) F(u(t))\, dr\, dt \qquad (1)$$

subject to the mixed condition

$$\sum_{j=0}^{m-1} [a_{ij} U^{(j)}(a) + b_{ij} U^{(j)}(b)] = \lambda_i \qquad (2)$$

where $m \geq 0; g(x)$ and $K(r,t)$ are analytic functions. $u(x)$ is the unknown to be determined, the upper limit x is a variable while a,b are constants.

2.0 METHODOLOGY

Consider approximate solution in the form

$$u_n(x) = \phi(x) A \qquad (3)$$

Where $\phi(x)$ is a Monic Chebyshev approximation polynomial, and

$$\phi(x) = [\phi_0(x) \ldots \phi_N(x)],$$

$$A = [a_0 \quad a_1 \quad \ldots \quad a_N]^T$$

Substituting (3) into (1) gives

$$\phi^{(m)}(x) A = \int_0^x \int_a^b K(r,t) \phi(t)\, dr\, dt \qquad (4)$$

Simplifying (4) gives a linear algebraic equation in the form

$$\tau(x) = g(x) \quad (5)$$

where

$$\tau(x) = \phi^{(m)}(x) - \int_0^x \int_a^b K(r,t)\phi(t)drdt$$

Collocating (5) at the standard collocation point x_i where

$$x_i = a + \frac{(b-a)}{N} i \quad (6)$$

gives

$$\tau(x_i)A = g(x_i) \quad (7)$$

$$\tau(x_i) = \begin{bmatrix} \tau_0(x_0) & \tau_1(x_0) & \cdots & \tau_N(x_0) \\ \tau_0(x_1) & \tau_1(x_1) & \cdots & \tau_N(x_1) \\ \vdots & \vdots & \ddots & \vdots \\ \tau_0(x_N) & \tau_1(x_N) & \cdots & \tau_N(x_N) \end{bmatrix}_{(N+1)\times(N+1)}$$

$$g(x_i) = [g(x_0) \;\; g(x_1) \;\; \cdots \;\; g(x_N)]^T_{(N+1)\times 1}$$

Considering the mixed condition, substituting (3) into (2) gives

$$\sum_{n=0}^{N} [a_{ij}\phi^m(a) + b_{ij}\phi^m(b)]A = \lambda_i$$

(8)

writing (8) in the form

$$U_i A = \lambda_i$$

(9)

Augmenting (7) with (9), gives a linear inconsistent equation

$$\gamma(x_i) = G(x_i)$$

(10)

where

$$\gamma(x_i) = \begin{bmatrix} \tau_0(x_0) & \tau_1(x_0) & \tau_2(x_0) & \cdots & \tau_N(x_0) \\ \tau_0(x_1) & \tau_1(x_1) & \tau_2(x_1) & \cdots & \tau_N(x_1) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \tau_0(x_N) & \tau_1(x_N) & \tau_2(x_N) & \cdots & \tau_N(x_N) \\ U_{00} & U_{01} & U_{02} & \cdots & U_{0N} \\ U_{10} & U_{11} & U_{12} & \cdots & U_{1N} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ U_{m-10} & U_{m-11} & U_{m-12} & \cdots & U_{m-1N} \end{bmatrix}_{[(N+1)+m-1]}$$

$$G(x_i) = [g(x_i)\lambda_i]^{\mathrm{T}}$$

$$\gamma^*(x_i)A = G^*(x_i)$$

(11)

Where,

$\gamma^*(x_i) = \gamma(x_i)^{\mathrm{T}}\gamma(x_i)$ and

$G^*(x_i) = \gamma(x_i)^{\mathrm{T}} g(x_i)$

$\gamma^*(x_i)$ is an $(N+1)\times(N+1)$ symmetric and positive definite matrix. Making A the subject in (9) which is then substituted into (2) gives the required approximate solution which is solved using conjugate gradient method.

3.0 NUMERICAL ILLUSTRATIONS

Example 1 We consider the first order linear mixed integro-differential equation given by

$$u'(x) = \frac{5}{4}x^2 + 8x + \int_0^x\int_0^1 (1-rt)u(t)dtdr$$

subject to the initial condition,

$$u(0) = 2,$$

the exact solution is

$$(x) = 2 + 6x^2$$

which is the exact solution.

Using $N = 2$ for illustration

$$\phi(x) = \begin{bmatrix} 1 & x & x^2 & -\frac{1}{2} \end{bmatrix}$$

$$A = \begin{bmatrix} a_0 & a_1 & a_3 \end{bmatrix}^T$$

$$g(x) = \frac{5}{4}x^2 + 8x$$

$$\tau(x) = \begin{bmatrix} \frac{x(x-4)}{4} & \frac{x(x-3)}{6} + 1 & \frac{13x}{6} \end{bmatrix}$$

$$x_i = \begin{bmatrix} 0 & \frac{1}{2} & 1 \end{bmatrix}$$

$$\tau(x_i) = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-7}{16} & \frac{19}{24} & \frac{13}{12} \\ \frac{-3}{4} & \frac{2}{3} & \frac{13}{6} \end{bmatrix}$$

$$G(x_i) = \begin{bmatrix} 0 & \frac{16}{19} & \frac{37}{4} & 2 \end{bmatrix}^T$$

$$\gamma^*(x) = \begin{bmatrix} \frac{449}{256} & \frac{-325}{384} & -\frac{499}{192} \\ -\frac{325}{384} & \frac{1193}{576} & \frac{221}{96} \\ -\frac{1747}{256} & \frac{3679}{384} & \frac{4553}{192} \end{bmatrix}$$

$$G^*(x_i) = \begin{bmatrix} -\frac{1747}{256} & \frac{3679}{384} & \frac{4553}{192} \end{bmatrix}^T$$

Using the conjugate residue method, we obtained the values for the unknown constant A as

$$A = \begin{bmatrix} 5 & 0 & 6 \end{bmatrix}^T$$

Substituting the values of A and $\phi(x)$ into (3) we get

$$u_n(x) = 2 + 6x^2$$

which is the exact solution.

Example 2: Consider the second order linear mixed integro-differential equations

$$u''(x) = -15x + \int_0^x \int_0^1 rtu(t)dtdr$$

subject to the initial condition

$u(0) = 1, \; u'(0) = 0$

with the exact solution

$$u(x) = 1 - \frac{5}{2}x^3$$

Using $N = 3$ for the illustration,

$$x_i = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} & 1 \end{bmatrix}$$

$$\phi(x) = \begin{bmatrix} 1 & x & x^2 - \frac{1}{2} & x\left(x^2 - \frac{1}{2}\right) - \frac{x}{4} \end{bmatrix}$$

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 \end{bmatrix}^T$$

$$g(x) = -15x$$

$$\tau(x) = \begin{bmatrix} \frac{-x^2}{4} & \frac{-x^2}{6} & 2 & \frac{x^2}{40} + 6x \end{bmatrix}$$

$$\tau(x_i) = \begin{bmatrix} 0 & 0 & 2 & 0 \\ \frac{-1}{36} & \frac{-1}{54} & 2 & \frac{721}{360} \\ \frac{-1}{9} & \frac{-2}{27} & 2 & \frac{361}{90} \\ \frac{-1}{4} & \frac{-1}{6} & 2 & \frac{241}{40} \end{bmatrix}$$

$$g(x) = \begin{bmatrix} 0 & -5 & -10 & -15 \end{bmatrix}^T$$

$$\gamma(x_i) = \begin{bmatrix} 0 & 0 & 2 & 0 \\ \frac{-1}{36} & \frac{-1}{54} & 2 & \frac{721}{360} \\ \frac{-1}{9} & \frac{-2}{27} & 2 & \frac{361}{90} \\ \frac{-1}{4} & \frac{-1}{6} & 2 & \frac{241}{40} \\ 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 1 & 1 & \frac{-3}{4} \end{bmatrix}$$

$$G(x_i) = \begin{bmatrix} 0 & \frac{16}{19} & \frac{37}{4} & 2 & 1 & 0 \end{bmatrix}^T$$

$$\gamma^*(x_i) = \begin{bmatrix} \frac{697}{648} & \frac{49}{972} & \frac{-23}{18} & \frac{-13009}{6480} \\ \frac{49}{972} & \frac{1507}{1458} & \frac{-14}{27} & \frac{-20299}{9729} \\ \frac{-23}{18} & \frac{-14}{27} & \frac{65}{4} & \frac{2167}{90} \\ \frac{-13009}{6480} & \frac{-20299}{9729} & \frac{2167}{90} & \frac{3691219}{64800} \end{bmatrix}$$

$$G^*(x_i) = \begin{bmatrix} 6 & \frac{10}{3} & \frac{-121}{2} & \frac{-281}{2} \end{bmatrix}^T$$

Using conjugate residue method, we obtained values for the unknown constant A as

$$A = \begin{bmatrix} 1 & \frac{-15}{8} & 0 & \frac{-5}{2} \end{bmatrix}^T$$

Substituting for A and $\phi(x)$ into (3) gives

$$u_n(x) = 1 - \frac{5}{2}x^3$$

which is the exact solution.

Example 3 We consider the second order linear mixed Integro-differential equation

$$u''(x) = 2 + 6x - \frac{77}{120}x^2 + \int_0^x \int_0^1 rtu(t)\,dtdr$$

subject to the initial condition

u(0) = 1; u0 (0) = 1

with the exact solution

$$u(x) = 1 + x + x^2 + x^3$$

using $N = 3$; for the illustration with the standard collocation points

$$x_i = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} & 1 \end{bmatrix}$$

$$\phi(x) = \begin{bmatrix} 1 & x & x^2 - \frac{1}{2} & x(x^2 - \frac{1}{2}) - \frac{x}{4} \end{bmatrix}$$

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 \end{bmatrix}^T$$

$g(x) = 2 + 6x - \frac{77}{120}x^2$

$\tau(x) = \begin{bmatrix} -\frac{x^2}{4} & -\frac{x^2}{6} & 2 & \frac{x^2}{40} + 6x \end{bmatrix}$

$$\tau(x_i) = \begin{bmatrix} 0 & 0 & 2 & 0 \\ -\frac{1}{36} & -\frac{1}{54} & 2 & \frac{721}{360} \\ -\frac{1}{9} & -\frac{2}{27} & 2 & \frac{361}{90} \\ -\frac{1}{4} & -\frac{1}{6} & 2 & \frac{241}{40} \end{bmatrix}$$

$g(x_i) = \begin{bmatrix} 2 & \frac{4243}{1080} & \frac{1543}{270} & \frac{883}{120} \end{bmatrix}^T$

$$\gamma(x_i) = \begin{bmatrix} 0 & 0 & 2 & 0 \\ -\frac{1}{36} & -\frac{1}{54} & 2 & \frac{721}{360} \\ -\frac{1}{9} & -\frac{2}{27} & 2 & \frac{361}{90} \\ -\frac{1}{4} & -\frac{1}{6} & 2 & \frac{241}{40} \\ 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 1 & -\frac{3}{4} \end{bmatrix}$$

$G(x) = \begin{bmatrix} 2 & \frac{4243}{1080} & \frac{1543}{270} & \frac{883}{120} & 1 & 1 \end{bmatrix}^T$

$$\gamma^*(x) = \begin{bmatrix} \frac{697}{648} & \frac{49}{972} & -\frac{23}{18} & -\frac{13009}{6480} \\ \frac{49}{972} & \frac{1507}{1458} & -\frac{14}{27} & -\frac{20299}{9729} \\ -\frac{23}{18} & -\frac{14}{27} & \frac{65}{4} & \frac{2167}{90} \\ -\frac{13009}{6480} & -\frac{20299}{9729} & \frac{2167}{90} & \frac{3691219}{64800} \end{bmatrix}$$

$G^*(x_i) = \begin{bmatrix} -\frac{30787}{19440} & -\frac{21067}{29160} & \frac{5063}{135} & \frac{14458507}{19440} \end{bmatrix}^T$

$A = \begin{bmatrix} \frac{3}{2} & \frac{7}{4} & 1 & 1 \end{bmatrix}$

$u_n(x) = 1 + x + x^2 + x^3$

which is the exact results.

Example 4 Consider the first order linear mixed intgro-differential equation

$u'(x) = 1 - 2x - \frac{43}{15}x^2 + \int_0^x \int_1^1 (1 - rt)\, u(t)\, dt dr$

subject to the initial condition

u(0) = 1

with the exact solution

$u(x) = 1 + x - x^3$

we give illustration to example 4 using $N = 3$ standard collocation points

$x_i = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} & 1 \end{bmatrix}$

$\phi(x) = \begin{bmatrix} 1 & x & x^2 - \frac{1}{2} & x(x^2 - \frac{1}{2}) - \frac{x}{4} \end{bmatrix}$

$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 \end{bmatrix}$

$g(x) = 1 - 2x - \frac{43}{15}x^2$

$\tau(x) = \begin{bmatrix} -2x & \frac{x^2}{3} + 1 & \frac{7x}{3} & \frac{59x^2}{20} - \frac{3}{4} \end{bmatrix}$

$$\tau(x_i) = \begin{bmatrix} 0 & 1 & 2 & -\frac{3}{4} \\ -\frac{2}{3} & \frac{28}{27} & \frac{7}{9} & -\frac{19}{45} \\ -\frac{4}{3} & \frac{31}{27} & \frac{14}{9} & \frac{101}{180} \\ -2 & \frac{4}{3} & \frac{7}{3} & \frac{11}{5} \end{bmatrix}$$

$g(x_i) = \begin{bmatrix} 1 & \frac{2}{135} & -\frac{217}{135} & -\frac{58}{15} \end{bmatrix}^T$

$$\gamma(x_i) = \begin{bmatrix} 0 & 1 & 2 & -\frac{4}{3} \\ -\frac{2}{3} & \frac{28}{27} & \frac{7}{9} & -\frac{19}{45} \\ -\frac{4}{3} & \frac{31}{27} & \frac{14}{9} & \frac{101}{180} \\ -2 & \frac{4}{3} & \frac{7}{3} & \frac{11}{5} \\ 1 & 0 & -\frac{1}{2} & 0 \end{bmatrix}$$

$G(x_i) = \begin{bmatrix} 1 & \frac{2}{135} & -\frac{217}{135} & -\frac{58}{15} & 1 \end{bmatrix}$

$$\gamma^*(x_i) = \begin{bmatrix} \frac{65}{9} & -\frac{44}{9} & -\frac{419}{54} & -\frac{73}{151} \\ -\frac{44}{9} & \frac{3770}{729} & \frac{154}{27} & \frac{5807}{2430} \\ -\frac{419}{54} & \frac{154}{27} & \frac{2825}{324} & \frac{511}{901} \\ -\frac{73}{151} & \frac{5807}{2430} & \frac{511}{901} & \frac{95509}{16200} \end{bmatrix}$$

$$G^*(x_i)=\begin{bmatrix}\frac{163}{15} & -\frac{21818}{3645} & -\frac{1081}{90} & -\frac{123503}{12150}\end{bmatrix}^T$$

$$A=\begin{bmatrix}1 & \frac{1}{4} & 0 & -1\end{bmatrix}$$

$$u_n(x)=1+x-x^3$$

which is the exact result.

Example 5 Consider the linear mixed Fredholm-Volterra integral equation with the exact solution

$$u(x)=6+9x+2x^2-2x^3+\int_0^x\int_{-1}^1(rt^2+r^2t)u(t)dtdr$$

With the exact solution

$$u(x)=6+9x+5x^2$$

using $N=3$ for the illustration, with standard collocation points.

$$x_i=\begin{bmatrix}0 & \frac{1}{3} & \frac{2}{3} & 1\end{bmatrix}$$

$$\phi(x)=\begin{bmatrix}1 & x & x^2-\frac{1}{2} & x(x^2-\frac{1}{2})-\frac{3}{4}\end{bmatrix}$$

$$A=\begin{bmatrix}a_0 & a_1 & a_2 & a_3\end{bmatrix}$$

$$g(x)=6+9x+2x^2-2x^3$$

$$\tau(x)=\begin{bmatrix}1-\frac{x^2}{3} & x-\frac{x^3}{6} & \frac{29x^2}{30}-\frac{1}{2} & x\left(x^2-\frac{1}{2}\right)-\frac{x}{4}+\frac{x^3}{30}\end{bmatrix}$$

$$\tau(x_i)=\begin{bmatrix}0 & 1 & -\frac{1}{2} & 0\\ \frac{26}{27} & \frac{79}{243} & -\frac{53}{135} & -\frac{343}{1620}\\ \frac{23}{27} & \frac{146}{243} & -\frac{19}{270} & -\frac{157}{810}\\ \frac{2}{3} & \frac{7}{9} & \frac{7}{15} & \frac{17}{60}\end{bmatrix}$$

$$g(x_i)=\begin{bmatrix}6 & \frac{247}{27} & \frac{332}{27} & 15\end{bmatrix}^T$$

$$\gamma^*(x_i)=\begin{bmatrix}\frac{2258}{729} & \frac{2938}{2187} & -\frac{457}{729} & -\frac{1313}{7290}\\ \frac{2938}{2187} & \frac{63278}{69049} & \frac{2111}{10935} & \frac{1381}{393661}\\ -\frac{457}{729} & \frac{2111}{10935} & \frac{22849}{36450} & \frac{16693}{72900}\\ -\frac{1313}{7290} & \frac{1381}{393661} & \frac{16693}{72900} & \frac{213463}{1312200}\end{bmatrix}$$

$$G^*(x_i)=\begin{bmatrix}\frac{2868}{81} & \frac{144530}{6661} & -\frac{37}{81} & -\frac{1537}{21870}\end{bmatrix}^T$$

$$A=\begin{bmatrix}\frac{17}{2} & 9 & 5 & 0\end{bmatrix}$$

$$u_n(x)=5x^2+9x+6$$

which is the exact solution

Example 6 Considering the mixed linear Fredholm-Volterra integral equation

$$u(x)=2+6x+\frac{15}{2}x^2+\int_0^x\int_0^1(r-t)u(t)dtdr$$

with the exact solution as

$$u(x)=2+6x+12x^2$$

we give illustration using $N=3$; with the standard collocation points

$$x_i=\begin{bmatrix}0 & \frac{1}{3} & \frac{2}{3} & 1\end{bmatrix}$$

$$\phi(x)=\begin{bmatrix}1 & x & x^2-\frac{1}{2} & x(x^2-\frac{1}{2})-\frac{x}{4}\end{bmatrix}$$

$$A=\begin{bmatrix}a_0 & a_1 & a_2 & a_3\end{bmatrix}^T$$

$$g(x)=\begin{bmatrix}2 & \frac{653}{4216} & \frac{145}{54} & -\frac{1}{8}\end{bmatrix}$$

$$\tau(x)=\begin{bmatrix}1-\frac{x(x-1)}{2}, & x-\frac{x(3x-4)}{12}, & \frac{13x^2}{12}-\frac{1}{2}, & \frac{x(5x-4)}{80}-\frac{x}{4}+x\left(x^2-\frac{1}{2}\right)\end{bmatrix}$$

$$\tau(x_i)=\begin{bmatrix}0 & 1 & -\frac{1}{2} & 0\\ \frac{10}{9} & \frac{5}{12} & -\frac{41}{108} & -\frac{481}{2160}\\ \frac{10}{9} & \frac{7}{9} & -\frac{1}{54} & -\frac{113}{540}\\ 1 & \frac{13}{12} & \frac{7}{12} & \frac{21}{80}\end{bmatrix}$$

$$g(x_i)=\begin{bmatrix}2 & \frac{41}{6} & \frac{40}{3} & \frac{43}{2}\end{bmatrix}^T$$

$$\gamma^*(x_i)=\begin{bmatrix}\frac{362}{81} & \frac{781}{324} & -\frac{349}{972} & -\frac{1409}{6480}\\ \frac{781}{324} & \frac{1265}{648} & \frac{893}{1944} & \frac{1121}{38880}\\ -\frac{349}{972} & \frac{893}{1944} & \frac{4285}{5832} & \frac{9391}{38880}\\ -\frac{1409}{6480} & \frac{1121}{38880} & \frac{9391}{38880} & \frac{378577}{2332800}\end{bmatrix}$$

$$G^*(x_i) = \begin{bmatrix} \frac{2479}{54} & \frac{3943}{108} & \frac{2819}{324} & \frac{959}{720} \end{bmatrix}^T$$

$$A = \begin{bmatrix} 8 & 6 & 12 & 0 \end{bmatrix}$$

$$u_n(x) = 2 + 6x + x^2$$

4.0 CONCLUSION

A collocation method for the numerical solution of higher order mixed linear integral and Integro-differential equations subject to mixed condition based on Monic Chebyshev polynomial is presented in this paper. Numerical results show that the approach is efficient and simple to implement since all the approximate solutions are the same as the exact solution. All computations are done with the aid of program written in MATLAB (R2015a) and run on a PC (Elitebook 2570P).

5.0 CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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